

MATHEMATICS-I

Subject code: MA101BS

Regulations: R18-JNTUH

Class: I Year B. Tech CE & ME I Sem



Department of Science and Humanities

BHARAT INSTITUTE OF ENGINEERING AND TECHNOLOGY

Ibrahimpattam - 501 510, Hyderabad

MATHEMATICS-I (MA101BS)

I. COURSE OVERVIEW:

The students will improve their ability to think critically, to analyze a real problem and solve it using a wide array of mathematical tools. They will also be able to apply these ideas to a wide range of problems that include the Engineering applications.

II. PREREQUISITE:

1. Different types of matrices.
2. Differentiation and Integration.
3. Concepts of sequence and series.
4. Basic knowledge of calculation of basic formulas.
5. Basic knowledge of partial differentiation.

III. COURSE OBJECTIVE:

1.	Types of matrices and their properties.
2.	Concept of a rank of the matrix and applying this concept to know the consistency and solving the system of linear equations
3.	Concept of Eigen values and eigenvectors and to reduce the quadratic form to canonical form.
4.	Concept of Sequence
5.	Concept of nature of the series
6.	Geometrical approach to the mean value theorems and their application to the mathematical problems
7.	Evaluation of surface areas and volumes of revolutions of curves

8.	Evaluation of improper integrals using Beta and Gamma functions
9.	Partial differentiation, concept of total derivative
10.	Finding maxima and minima of function of two and three variables

IV.COURSE OUTCOMES:

S. No	Description	Bloom's Taxonomy Level
1.	Write the matrix representation of a set of linear equations and to analyse the solution of the system of equations	Knowledge, Analyze (Level 1,Level 4)
2.	Find the Eigen values and Eigen vectors	Knowledge (Level 1)
3.	Reduce the quadratic form to canonical form using orthogonal transformations	Knowledge, Analyze (Level 1,Level 4)
4.	Analyse the nature of sequence and series	Knowledge, Analyze (Level 1,Level 4)
5.	Solve the applications on the mean value theorems	Knowledge, Analyze (Level 1,Level 4)
6.	Evaluate the improper integrals using Beta and Gamma functions	Knowledge, Analyze (Level 1,Level 4)
7.	Find the extreme values of functions of two variables with/ without constraints	Knowledge, Analyze (Level 1,Level 4)

V. HOW PROGRAM OUTCOMES ARE ASSESSED:

Program Outcomes		Level	Proficiency assessed by
PO1	Anability to apply knowledge of computing, mathematical foundations, algorithmic principles, and computerscience and engineering theory in the modeling and design of computer-based system .world problems (fundamental engineering analysis skills)	1	Assignments and Tutorials.
PO2	An ability to design and conduct experiments, as well as to analyze and interpret data (information retrieval skills)	3	Assignments, Tutorials and Exams.

PO3	An ability to design, implement ,and evaluate a computer- An ability to design , implement, and evaluate a computer-based system, process, component, or program to meet desired needs, within realistic constraints such as economic, environmental, social, political, health and safety, manufacturability, and sustainability (Creative Skills) and sustainability (Creative Skills)	3	Assignments, Tutorials and Exams.
PO4	An ability to function effectively on multi-disciplinary teams (team work)	1	--
PO5	An ability to analyze a problem, identify, formulate and use the appropriate computing and engineering requirements for obtaining its solution (Engineering problem solving skills)	3	Assignments and Exams
PO6	An understanding of professional, ethical, legal, security and social issues and responsibilities (professional integrity)	1	--
PO7	An ability to communicate effectively both in writing and orally (speaking / writing skills)	1	--
PO8	The broad education necessary to analyze the local and global impact of computing and engineering solutions on individuals, organizations, and society (engineering impact assessment skills)	1	Assignments and Exams.
PO9	Recognition of the need for, and an ability to engage in continuing professional development and life-long learning (continuing education awareness)	3	Assignments and Exams
PO10	A Knowledge of contemporary issues (social awareness)	3	Assignments and Exams
PO11	An ability to use current techniques, skills, and tools necessary for computing and engineering practice (practical engineering analysis skills)	3	Assignments and Exams
PO12	An ability to apply design and development principles in the construction of software and hardware systems of varying complexity (software hardware interface)	--	--

1: Slight (Low)

2: Moderate (Medium)

3: Substantial (High)

4: None

VI. HOW PROGRAM SPECIFIC OUTCOMES ARE ASSESSED:

Program Specific Outcomes		Level	Proficiency assessed by
PSO1	UNDERSTANDING: Graduates will have an ability to understand, analyze and solve problems using basic mathematics and apply the techniques related to irrigation, structural design, etc.	3	Assignments, Tutorials and Exams.
PSO2	ANALYTICAL SKILLS: Graduates will have an ability to design civil structures, using construction components and to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety manufacturability and reliability and learn to work with multidisciplinary teams.	3	Projects
PSO3	BROADNESS: Graduates will have an exposure to various fields of engineering necessary to understand the impact of other disciplines on civil engineering blueprints in a global, economic, and societal context and to have necessary focus for postgraduate education and research opportunities at global level.	1	Guest Lectures

1: Slight (Low)

2: Moderate (Medium)

3: Substantial (High)

4: None

VII. SYLLABUS:

UNIT-I: Matrices

Matrices: Types of Matrices, Symmetric; Hermitian; Skew-symmetric; Skew-Hermitian; orthogonal matrices; Unitary Matrices; rank of a matrix by Echelon form and Normal form, Inverse of Non-singular matrices by Gauss-Jordan method; System of linear equations; solving system of Homogeneous and Non-Homogeneous equations. Gauss elimination method; Gauss Seidel Iteration Method.

UNIT-II: Eigen values and Eigen vectors

Linear Transformation and Orthogonal Transformation: Eigen values and Eigenvectors and

their properties: Diagonalization of a matrix; Cayley-Hamilton Theorem (without proof); finding inverse and power of a matrix by Cayley-Hamilton Theorem; Quadratic forms and Nature of the Quadratic Forms; Reduction of Quadratic form to canonical forms by Orthogonal Transformation

UNIT-III: Sequences & Series

Sequence: Definition of a Sequence, limit; Convergent, Divergent and Oscillatory sequences. Series: Convergent, Divergent and Oscillatory Series; Series of positive terms; Comparison test, p-test, D-Alembert's ratio test; Raabe's test; Cauchy's Integral test; Cauchy's root test; logarithmic test. Alternating series: Leibnitz test; Alternating Convergent series: Absolute and Conditionally Convergence.

UNIT-IV: Calculus

Mean value theorems: Rolle's theorem, Lagrange's Mean value theorem with their Geometrical Interpretation and applications, Cauchy's Mean value Theorem. Taylor's Series. Applications of definite integrals to evaluate surface areas and volumes of revolutions of curves (Only in Cartesian coordinates), Definition of Improper Integral: Beta and Gamma functions and their applications.

UNIT-V: Multivariable calculus (Partial Differentiation and applications)

Definitions of Limit and continuity. Partial Differentiation; Euler's Theorem; Total derivative; Jacobian; Functional dependence and independence, Maxima and minima of functions of two variables and three variables using method of Lagrange multipliers.

GATE SYLLABUS:

Differential Equations: Ordinary Differential Equations First order ordinary differential equations, existence and uniqueness theorems for initial value problems, systems of linear first order ordinary differential equations, linear ordinary differential equations of higher order with constant coefficients; linear second order ordinary differential equations with variable coefficients

Linear Systems of Equations:

Linear transformations and their matrix representations, rank; systems of linear equations, Hermitian, Skew Hermitian and unitary matrices, Jordan

Eigen values, Eigen Vectors:

Eigen values and eigenvectors, minimal polynomial, Cayley-Hamilton Theorem, diagonalization

Functions of several variables: Functions of several variables, maxima, minima

Partial Differential Equations:

Partial Differential Equations Linear and quasi-linear first order partial differential equations, method of characteristics; second order linear equations in two variables and their classification;

IES SYLLABUS:

Matrix: Matrix theory, Eigen values & Eigen vectors, system of linear equations

Differential Equations: Numerical methods for solution of non-linear algebraic equations and differential equations

Partial differential equations: Partial derivatives, linear, nonlinear and partial differential equations, initial and boundary value problems

VIII. LESSON PLAN-COURSE SCHEDULE:

Sessi on	Week No	Unit	TOPIC	Course learning outcomes	Reference
1.	1		Types of Matrices	Define Types of matrices	T1,T2,R3,R4
2.			Symmetric, Skew-symmetric and orthogonal matrices	Define real matrices	T1,T2,R3,R4
3.			Hermitian, Skew-Hermitian and	Define complex matrices	T1,T2,R3,R4

		1	Unitary Matrices		
4.			Rank of matrix with examples	Find rank of a matrix	T1,T2,R3,R4
5.			Echelon form with problems	Solve problems on echelon form	T1,T2,R3,R4
6.	2		Normal form with problems	Solve problems on normal form	T1,T2,R3,R4
7.			Inverse of Non-singular matrices by Gauss-Jordan method	Find inverse by Gauss-Jordan method	T1,T2,R3,R4
8.			System of linear equations	Solve problems	T1,T2,R3,R4
9.			Solving system of Homogeneous equations	Solve Homogeneous equations	T1,T2,R3,R4
10.			Solving system of Non-Homogeneous equations	Solve Non-Homogeneous equations	T1,T2,R3,R4
11.			Gauss elimination method	Solve problems using Gauss elimination method	T1,T2,R3,R4
12.	3		Gauss Seidel Iteration Method.	Solve problems using Gauss Seidel Iteration Method.	T1,T2,R3,R4
			*Applications of Arrangements and transformations (content beyond syllabus)	Understand applications	
			Mock Test – I		
UNIT – 2					
13.		2	Introduction of Eigen values and Eigen vectors	Define Eigen values and Eigen vectors	T1,T2
14.			Problems on Eigen values and Eigen vectors	Solve problems	T1,T2
15.	5		Eigen values, Eigen vectors and	Apply properties on Eigen	T1,T2

			their properties	values and Eigen vectors	
16.			Properties of Eigen values, Eigen vectors	Apply properties on Eigen values and Eigen vectors	T1,T2
17.			Diagonalization	Find Diagonalization	T1,T2
18			Problems Diagonalization	Solve problems	T1,T2
19			Cayley - Hamilton theorem and Problems on Inverse and powers of a matrix using Cayley	Solve problems	T1,T2
20.			Quadratic forms	Solve problems	T1,T2
21.			Nature of the Quadratic Forms	Find Nature of the Quadratic Forms	T1,T2
22.			Rank, Index and signature of the Quadratic forms	Find Rank, Index and signature of the Quadratic forms	T1,T2
23.	6		Reduction of Quadratic form to canonical forms by Orthogonal Transformation	Solve problems	T1,T2
24.		2	Reduction of Quadratic form to canonical forms by Orthogonal Transformation	Solve problems	T1,T2
			*Applications of significant of vectors (topic beyond syllabus)	Apply vectors	
			Tutorial / Bridge Class # 1		
UNIT – 3					
25.			Definition of a Sequence	Define sequence	T1,T2,R3,R4
26.	7		Limit, Convergent, Divergent and Oscillatory sequences	Define Limit, Convergent, Divergent and Oscillatory sequences	T1,T2,R3,R4
27.			Series: Convergent, Divergent and Oscillatory Series	Define series	T1,T2,R3,R4

28.	8	3	Series of positive terms, Comparison test, and p-test	Solve problems	T1,T2,R3,R4
29.			D-Alembert's ratio test and Raabe's test	Evaluate integral	T1,T2,R3,R4
30.			Cauchy's Integral test	Solve problems	T1,T2,R3,R4
31.			Cauchy's root test	Solve problems	T1,T2,R3,R4 T1,T2,R3,R4
32.			logarithmic test	Solve problems	T1,T2,R3,R4
			Tutorial / Bridge Class # 2		
I Mid Examinations					
33.	9	3	Alternating series and Leibnitz test	DefineAlternating series	T1,T2,R3,R4
34.			Alternating Convergent series	Solve problems	T1,T2,R3,R4
35.			Absolute Convergence.	Solve problems	T1,T2,R3,R4
36.			Conditionally Convergence.	Solve problems	T1,T2,R3,R4
			*Convergence and divergence of signals and systems (contents beyond the syllabus)	Understand application	
UNIT – 4					
37	10		Mean value theorems: Rolle's theorem	Apply Rolle's theorem	T1,T2,R3,R4
38			Lagrange's Mean value theorem with their Geometrical Interpretation	Apply Mean value theorem	T1,T2,R3,R4
39			Lagrange's Mean value theorem with their applications	Apply Mean value theorem	T1,T2,R3,R4
40			Cauchy's Mean value Theorem	Apply Cauchy's Mean value	T1,T2,R3,R4

				Theorem	
41	11		Taylor's Series	Apply Taylor's Series	T1,T2,R3,R4
42			Applications of definite integrals to evaluate surface areas of revolutions of curves in Cartesian coordinates	Solve problems	T1,T2,R3,R4
43			Applications of definite integrals to evaluate surface volumes of revolutions of curves in Cartesian coordinates	Solve problems	T1,T2,R3,R4
44		4	Applications of definite integrals to evaluate surface areas and volumes of revolutions of curves in Cartesian coordinates	Solve problems	T1,T2,R3,R4
			<i>Tutorial / Bridge Class # 3</i>		
45			Definition of Improper Integral: Beta function	Define Improper Integral	T1,T2,R3,R4
46			Gamma functions	Apply Gamma functions	T1,T2,R3,R4
47			Applications of Beta functions	Apply betafunctions	T1,T2,R3,R4
48			Applications of Gamma functions	Apply Gamma functions	T1,T2,R3,R4
	12		Applications of improper integrals and Signals and Systems, Linear Integrated Circuits and digital Signal Processing (content beyond the syllabus)	Know applications	
			<i>Mock Test - II</i>		
UNIT – 5					
49	13		Definitions of Limit and	Define Limit and continuity	T1,T2,R3,R4

			continuity		
50			Partial Differentiation	Define Partial Differentiation	T1,T2,R3,R4
51			Partial Differentiation	Define Partial Differentiation	T1,T2,R3,R4
52			Euler's Theorem	Apply Euler's Theorem	T1,T2,R3,R4
53			Total derivative and Jacobian	Solve problems	T1,T2,R3,R4
54		14	Functional dependence and independence	Solve problems	T1,T2,R3,R4
55			Functional independence	Solve problems	T1,T2,R3,R4
56			Maxima and minima of functions of two variables	Solve problems	T1,T2,R3,R4
57			Maxima and minima of functions of three variables	Solve problems	T1,T2,R3,R4
58		5	Maxima and minima of functions of two variables and three variables using method of Lagrange	Solve problems	T1,T2,R3,R4
59			Maxima and minima of functions of two variables and three variables using method of Lagrange	Solve problems	T1,T2,R3,R4
60		15	Maxima and minima of functions of two variables and three variables using method of Lagrange	Solve problems	T1,T2,R3,R4
			PDE applications in Computational Fluid Dynamics and Aerodynamics etc.(content beyond syllabus)	Know applications	
			<i>Tutorial / Bridge Class # 4</i>		
II Mid Examinations					

SUGGESTED BOOKS:**TEXT BOOK:**

1. A first course in differential equations with modeling applications by Dennis G. Zill, Cengage Learning publishers.
2. Higher Engineering Mathematics by Dr. B. S. Grewal, Khanna Publishers.

REFERENCE BOOKS

3. Advanced Engineering Mathematics by R.K. Jain & S.R.K. Iyengar, 3rd edition, Narosa Publishing House, Delhi.
4. Engineering Mathematics–I by T.K. V. Iyengar, B. Krishna Gandhi & Others, S. Chand.
5. Engineering Mathematics–I by D. S. Chandrasekhar, Prison Books Pvt. Ltd.
6. Engineering Mathematics–I by G. Shanker Rao & Others I.K. International Publications.
7. Advanced Engineering Mathematics with MATLAB, Dean G. Duffy, 3rd Edi, CRC Press Taylor & Francis Group.
8. Mathematics for Engineers and Scientists, Alan Jeffrey, 6th Edi, 2013, Chapman & Hall/ CRC
9. Advanced Engineering Mathematics, Michael Greenberg, Second Edition. Pearson Education.

IX. MAPPING COURSE OUTCOMES LEADING TO THE ACHIEVEMENT OF**PROGRAM OUTCOMES AND PROGRAM SPECIFIC OUTCOMES:**

Course Outcome	Program Outcomes (PO)												PSO1	PSO2
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12		
CO1	1	3	1	-	-	-	-	-	-	-	-	-	1	-
CO2	2	2	-	-	-	-	-	-	-	-	-	-	1	2
CO3	2	3	1	-	2	-	-	-	-	-	-	-	2	-
CO4	2	2	-	2	-	1	-	-	-	-	-	-	-	-
CO5	1	2	-	-	-	-	-	-	-	-	-	-	1	-
AVG	1.6	1.2	0.4	0.4	0.4	0.2	-	-	-	-	-	-	1	0.4

1: Slight(Low)**2: Moderate (Medium)****3: Substantial(High)****4 : None****QUESTION BANK: (JNTUH)**

DESCRIPTIVE QUESTIONS:**UNIT I****Short Answer Questions**

S.No	Question	Blooms taxonomy level	Course outcome
1	Define conjugate of a matrix.	Remember	1
2	If A is Hermitian matrix Prove that iA is skew- Hermitian matrix	Analyse	1
3	Prove that $\frac{1}{2} \begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary matrix.	Understand	1
4	Find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.	Evaluate	1
5	Find the Skew- symmetric part of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$	Evaluate	1

Long Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Express the matrix $\begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$ as the sum of Hermitian and Skew-Hermitian matrix.	Understand	1
2	Find the rank of the matrix $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$	Evaluate	1
3	Find the rank of the matrix $\begin{bmatrix} 1 & 0 & -4 & 5 \\ 2 & -1 & 3 & 0 \\ 8 & 1 & 0 & -7 \end{bmatrix}$	Evaluate	1

4	Find a and b such that rank of $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is 3.	Evaluate	1
5	Find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$	Evaluate	1
6	Given that $A = \begin{bmatrix} 0 & 1-2i \\ -1-2i & 0 \end{bmatrix}$ show that $(I-A)(I+A)^{-1}$ is unitary matrix.	Analyze	1
7	For what value of K such that the matrix $\begin{pmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{pmatrix}$ has rank 3	Analyze	1
8	Find rank by reducing to Normal form of matrix	Evaluate	1
9	Reduce the matrix A to its normal form where $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ and hence find the rank	Evaluate	1
10	Solve the system of equations $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$.	Analyze	1

UNIT II

Short Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Find the Eigen values of the matrix $\begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$	Evaluate	2
2	State Cayley- Hamilton Theorem	Remember	2
3	Find the Eigen values of the matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$	Evaluate	2
4	Identify the nature of the quadratic form $3x^2+3y^2+3z^2+2xy+2xz-2yz$.	Remember	3

5	If 2,3,4 are the Eigen values of A then find the Eigen values of adjA	Evaluate	2
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Long Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Diagonalize the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$ and find A^4	Analyze	2
2	Prove that the Eigen Values of Real symmetric matrix are Real.	Analyse	2
3	Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and find A^{-1} & A^4	Evaluate	2
4	Prove that the sum of the Eigen Values of a matrix is equal to its trace and Product of the Eigen Values is equal to its determinant.	Analyze	2
5	Find the Eigen values and Eigen vectors of Hermitian matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$	Evaluate	2
6	Prove that Eigen values of a skew- Hermitian matrix are either zero or purely imaginary.	Analyse	2
7	Express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$	Understand	2
8	Reduce to sum of squares, the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$ and find the rank, index and signature	Understand	3
9	Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ verify Cayley-Hamilton theorem hence find A^{-1} and A^4	Apply	2
10	Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ by similarity transformation and hence find A^4 .	Apply	2

UNIT III

Short Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Define sequence and series.	Remember	4
2	State cauchy's n^{th} root test.	Remember	4
3	Define absolute convergence and conditional convergence.	Remember	4
4	State logarithmic test .	Remember	4
5	Show that the sequence $\frac{1}{n}$ convergent.	Understand	4

Long Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Test for the convergence of the series $\sum \left(\frac{nx}{1+n}\right)^n$	Understand	4
2	Test for the convergence of the series $\sum \left(1 - \frac{1}{n}\right)^{-n^2}$	Understand	4
3	Test for the convergence of the series $\sum \frac{x^{2n}}{(n+1)\sqrt{n}}$	Understand	4
4	Test for the convergence of the series $\frac{2}{1} - \frac{2.5}{1.5} - \frac{2.5.8}{1.5.9} - \frac{2.5.8.11}{1.5.9.13} - \dots$	Understand	4
5	Test for the convergence of summation of $\frac{1}{\sqrt{n}-\sqrt{n+1}}$	Understand	4
6	Test for the boundedness for the sequence $\frac{1}{n^2}$	Understand	4
7	Test for the convergence of summation of $\frac{1}{\sqrt{n}+\sqrt{n+1}}$	Understand	4
8	Test whether the series $\sum (-1)^{n+1}(\sqrt{n+1} - \sqrt{n})$ is absolute convergent or conditional convergent.	Understand	4
9	Test whether the series $\sum (-1)^{n-1}\left(\frac{1}{n}\right)$ is absolute convergent or conditional convergent.	Understand	4
10	Test whether the series $\sum (-1)^{n-1}\left(\frac{1}{n^2}\right)$ is absolute convergent or conditional convergent.	Understand	4

UNIT IV

Short Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	What is the value of c in Rolle's theorem for $f(x)=\sin x/e^x$ in $(0,\pi)$	Analyse	5
2	What is the value of c in cauchy's mean value theorem for the function $f(x)=x^2, g(x)=x^3$ in $(1,2)$.	Analyse	5
3	Define Beta and Gamma function.	Remember	6
4	Define Lagrange's mean value Theorem.	Remember	5
5	Define Cauchy's Mean Value Theorem.	Remember	5

Long Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Verify Rolle's theorem for the function $f(x) = \sin x/e^x$ or $e^{-x} \sin x$ in $[0,\pi]$	Apply	5
2	Verify Rolle's theorem for the functions $\log \left(\frac{x^2 + ab}{x(a+b)} \right)$ in $[a, b]$, $a > 0, b > 0$,	Apply	5
3	Verify whether Rolle's Theorem can be applied to the following functions in the intervals. i) $f(x) = \tan x$ in $[0, \pi]$ and ii) $f(x) = 1/x^2$ in $[-1,1]$	Apply	5
4	Using Rolle's Theorem, show that $g(x) = 8x^3 - 6x^2 - 2x + 1$ has a zero between 0 and 1.	Apply	5
5	Verify Lagrange's Mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in $[0,4]$	Apply	5
6	If $a < b$, P.T $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using Lagrange's Mean value theorem. Deduce the following. (i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$ ii). $\frac{5\pi + 4}{20} < \tan^{-1} 2 < \frac{\pi + 2}{4}$	Apply	5
7	Show that for any $x > 0$, $1 + x < e^x < 1 + xe^x$.	Apply	5
8	Prove the relation between Beta and Gamma functions.	Apply	6
9	Find c of Cauchy's mean value theorem for	Apply	5

	$f(x) = \sqrt{x}$ & $g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$ where $0 < a < b$		
10	Verify Cauchy's Mean value theorem for $f(x) = e^x$ & $g(x) = e^{-x}$ in $[3, 7]$ and find the value of c	Apply	5

UNIT V

Short Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Expand $\log(1 + x)$ in powers of x .	Remember	7
2	If $x + y + z = u, y + z = uv, z = uvw$ then find Jacobian of x, y, z .	Understand	7
3	Find the maximum and minimum values of $f(x, y) = x^3 + y^3 - 3axy$	Understand	7
4	Find the maximum and minimum values of $\sin x + \sin y + \sin(x + y)$	Understand	7
5	The minimum value of $x^2 + y^2 + z^2$ given that $xyz = a^3$	Understand	7

Long Answer Questions

S.No	Question	Blooms taxonomy level	Course outcome
1	Prove that $e^x \cos x = 1 + x - \frac{2x^3}{3!} \dots$	Understand	7
2	If u and v are functions of x and y defined by $x = u + e^{-v} \sin u, y = v + e^{-v} \cos u$ prove that: $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$.	Understand	7
3	For spherical polar co-ordinates $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$, show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$.	Understand	7
4	Discuss the maxima, minima of $x^2 + y^2 + z^2$ where x, y, z are connected by $xyz = a^3$.	Remember	7
5	Find the volume of the largest parallelepiped that can be inscribed in	Understand	7

9. The matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is _____. Ans: c
 a) Singular b) Symmetric c) Skew-symmetric d) None
10. If A, B are two matrices of same order, then $\text{rank}(A+B)$ is _____. Ans: c
 a) $= \text{rank } A + \text{rank } B$ b) $< \text{rank } A + \text{rank } B$
 c) $\leq \text{rank } A + \text{rank } B$ d) $\geq \text{rank } A + \text{rank } B$

UNIT II

1. The Eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ are _____. Ans: (a)
 a) 1, 2, 3 b) -1, 0, 1 c) 0, 0, 1 d) 1, 2, -3
2. The characteristic equation of $\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$ is _____. Ans: (a)
 a) $\lambda^2 - 6\lambda + 3 = 0$ b) $\lambda^2 - 6\lambda + 3 = 0$
 c) $\lambda^2 - 6\lambda + 3 = 0$ d) $\lambda^2 - 6\lambda + 3 = 0$
3. The λ is an eigen value of a square matrix, then $\frac{1}{\lambda}$ is an Eigen value of _____. Ans: (d)
 a) A^2 b) A c) A^T d) A^{-1}
4. If the sum of the Eigen values of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & k \end{bmatrix}$ is 5, then the value of k is _____.
 Ans: (d)
 a) 0 b) 2 c) -2 d) -1
5. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ then the Eigen vector corresponding to its Eigen value 3 is _____. Ans: (b)
 a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
6. The Eigen value of a matrix is 0, when the matrix is _____. Ans: (d)
 a) Non-singular b) Orthogonal c) Skew-symmetric d) Singular
7. If α is an eigen vector of the matrix A corresponding to the eigen value k , then _____.
 Ans: (d)
 a) $A\alpha = k^2\alpha$ b) $A\alpha = k\alpha$ c) $A = k^2\alpha$ d) $A\alpha = k\alpha$
8. Every scalar matrix is _____. Ans: (a)
 a) Diagonal matrix b) Symmetric matrix c) skew-symmetric matrix d) None
9. If a square matrix A of order 2 has eigen vectors $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then A is a _____. Ans: (c)
 a) Skew-symmetric matrix b) scalar matrix
 c) diagonalizable matrix d) singular matrix
10. If λ is an eigen value of A , then λ^4 is an eigen value of _____. Ans: (a)
 a) A^4 b) A^2 c) A^3 d) A

UNIT III

- Convergence of the series $\frac{2}{1} - \frac{2.5}{1.5} - \frac{2.5.8}{1.5.9} - \frac{2.5.8.11}{1.5.9.13} - \dots$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (a)
- Convergence of summation of $\frac{1}{\sqrt{n} + \sqrt{n+1}}$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (b)
- Convergence of the series $\sum \left(1 - \frac{1}{n}\right)^{-n^2}$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (a)
- Convergence of the sequence $\left(\frac{n+1}{n}\right)$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (a)
- Convergence of the sequence $\left(\frac{n}{n^2+1}\right)$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (a)
- Convergence of the series $1^2 + 2^2 + 3^2 + 4^2 + \dots$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (b)
- Convergence of the series $\sum \frac{1}{n \log n}$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (b)
- Convergence of the series $\frac{1}{1.4} + \frac{1}{2.5} + \frac{1}{3.6} + \dots$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (a)
- Convergence of the series $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (b)
- Convergence of the series $\sum \frac{1}{2n+3}$ is
a) Convergent b) Divergent c) Oscillatory d) None Ans: (b)

UNIT IV

- The value of c of rolles theorem for $f(x) = \frac{\sin x}{e^x}$ in $(0, \pi)$ is _____. Ans. b
a) π b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$
- Using which mean value theorem we can calculate approximately the value of $65^{1/6}$ in an easier way
Ans. a
a) lagrange b) rolles c) cauchys d) none
- Find c using LMVT for $f(x) = \log x$ in $[1, e]$ Ans. c
a) e+1 b) 0 c) e-1 d) 1

4. Find c using CMVT for $f(x)=x^2$, $g(x)=x^3$ in $[1,2]$ Ans.d
 a) 14/5 b) 12/7 c) 12/5 d) 14/3
5. Gamma of one is Ans. c
 a) zero b) two c) one d) none
6. The value of c of rolles theorem for $f(x)=\tan x$ in $(0,\pi)$ is Ans.c
 a) 2 b) 1 c) Not applicable d) 3
7. Find c using CMVT for $f(x)=\sin x$, $g(x)=\cos x$ in $[a, b]$ Ans.d
 a) a-b b) a+b c) (a-b)/2 d) (a+b)/2
8. The value of c of rolles theorem for $f(x)=x+\frac{1}{x}$ in $(\frac{1}{2},2)$ is Ans.b
 a) 0 b) 1 c) 2 d) 3
9. Find c using LMVT for $f(x)=x^2 - 3x + 2$ in $[-2,3]$ Ans.a
 a) $\frac{1}{2}$ b) 0 c) 1 d) none
10. Gamma $\frac{1}{2}$ is ----- Ans.c
 a) 0 b) 1 c) $\sqrt{\pi}$ d) none

UNIT V

1. If $u=yz/x$, $v=zx/y$, $w=xy/z$, then $\partial(u,v,w)/\partial(x,y,z) = \underline{\hspace{2cm}}$. Ans. d
 a) 1 b) 2 c) 3 d) 4
2. If $u=x^2-2y$, $v = x+y+z$, $w = x-2y+3z$, then $\partial(u,v,w)/\partial(x,y,z) = \underline{\hspace{2cm}}$. Ans. b
 a) $5x+3$ b) $10x+4$ c) 5 d) 0
3. If $u=x + y + z$, $uv=y + z$, $uvw=z$, then $\partial(x,y,z)/\partial(u,v,w) = \underline{\hspace{2cm}}$. Ans.c
 a) uv b) u+v c) u^2v d) 1
4. If the functions u, v, w are said to be functionally dependent then $\partial(u,v,w)/\partial(x,y,z)$ is . Ans.a
 a) 0 b) 1 c) 2 d) 3
5. If the functions u, v, w are said to be functionally Independent then $\partial(u,v,w)/\partial(x,y,z)$ is . Ans. b
 a) Not equal to zero b) equal to zero c) undefined d) none
6. If $u=u(x,y)$ & $v=v(x,y)$, then $\partial(u,v)/\partial(x,y) \times \partial(x,y)/\partial(u,v) = \underline{\hspace{2cm}}$. Ans.b
 a) 0 b) 1 c) 2 d) 3
7. If $u=x+y+z$, $v=xy+yz+zx$, $w=x^2+y^2+z^2$ are functionally dependent then the relation between them is . Ans.d

- a) $w=u+v$ b) 1 c) $w=4v$ d) $w^2=2u+v$
8. If $u=x+y+z$, $v=x^3+y^3+z^3-3xyz$, $w=x^2+y^2+z^2-xy-yz-zx$ are functionally dependent then the relation between them is _____. Ans.c
- a) $w=u+v$ b) $w=uv$ c) $uw=v$ d) none
9. If $u=x+y$, $w=x^3+y^3+z^3$, $v=x^2+y^2+z^2-2xy-2yz-2zx$ are functionally dependent then the relation between them is _____. Ans.a
- a) $u+w=v$ b) 0 c) 1 d) 2
10. If $u=(x-y)/x+y$, $v=xy/(x+y)^2$ are functionally dependent then the relation between them is _____. Ans.a
- a) $u^2+4v=1$ b) $u=v$ c) $u+v=0$ d) none

GATE:

1. The value of the determinant $\begin{vmatrix} 2000 & 2001 & 2002 \\ 2003 & 2004 & 2005 \\ 2006 & 2007 & 2008 \end{vmatrix}$ is _____. Ans: a
- a) 0 b) 1 c) 2 d) 3
2. If ω is a cube root of unity, then, $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} = ____$ Ans: c
- a) 1 b) 3 c) 0 d) 2
3. The system of equations $x + y + z = a$, $3x - ay - 2z = b$, $5x - 7y = c$ has a solution if _____ Ans: a
- a) $b = -2a + c$ b) $b = 2a + c$ c) $b = 2a - c$ d) None
4. The equations $2x - y - z = 0$, $x + y = z = 0$, $3x + 6y + 8z = 0$ have _____ Ans: a
- a) Unique solution b) infinitely many solution c) No solution d) None
5. The conjugate of a matrix will be obtained by _____ Ans: c
- a) Interchanging rows and columns
b) taking transpose of the matrix
c) replacing the conjugate of the each and every element in the matrix
6. A square matrix A is said to be Hermitian if _____ Ans: a
- a) $A^\theta = A$ b) $A^\theta = -A$ c) $A^T = A$ d) $A^T = -A$
7. The system of equation $AX = B$, where $B = 0$ is _____ Ans: b
- a) Not consistent b) Always consistent
c) consistent only if $\rho(A|B) = \rho(A)$ d) need not be consistent
8. Given that the three matrices $\begin{bmatrix} 1 & x & 0 \\ 0 & y & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \\ x & 1 & 0 \\ 0 & y & 0 \end{bmatrix}$, $\begin{bmatrix} x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are non-singular matrices with equal determinants, then the values of x and y are _____. Ans: b
- a) $x = 1, y = -1$ b) $x = 1, y = 1$ c) $x = -1, y = 1$ d) None

9. The eigen values of $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ are _____ Ans: (b)
- a) 1, 3 b) 2, 4 c) -3, -4 d) 1, -4
10. If A is a square matrix and its characteristic equation is $\lambda^2 - 2\lambda - 8 = 0$, then ____
- Ans: (d)
- a) $A^{-1} = \frac{1}{4}(A^2 - 2A)$ b) $A^{-1} = \frac{1}{8}(A^2 - 2A)$
 c) $A^{-1} = \frac{1}{4}(2A - A^2)$ d) $A^{-1} = \frac{1}{8}(A - 2I)$
11. The eigen values of I_3 are _____ Ans: (a)
- a) 1, 1, 1 b) 0, 0, 0 c) 3, 3, 3 d) 2, 2, 2
12. The sum and product of the eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are _____ Ans: (b)
- a) 7, 5 b) 7, -36 c) 9, 7 d) 7, 9
13. If all the eigen values of a square matrix A are non zero, then A is _____ Ans: (b)
- a) singular b) Non-singular c) symmetric d) skew-symmetric
14. The quadratic form corresponding to the matrix $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ is _____ Ans: d
- a) $ax^2 + by^2 + cz^2$ b) $ax^2 + by^2 + cz^2 + hxy + fyz + gzx$
 c) $2hxy + 2fyz + 2gzx$ d) $ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx$
15. A quadratic form is positive definite when _____ Ans: b
- a) all the eigen values are ≥ 0 and atleast one eigen value is zero
 b) all the eigen values are positive
 c) some eigen values are positive
 d) all the eigen values are negative
16. The index and signature of the quadratic form $2x^2 + 2y^2 + 2z^2 + 2xy$ are _ Ans: b
- a) 3, 1 b) 3, 3 c) 3, 2 d) 2, 1
17. The rank of the quadratic form $2x_1x_3 + 6x_1x_3 - 4x_2x_3$ is _____ Ans: c
- a) 1 b) 2 c) 3 d) 0
18. The eigen values of $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are _____ Ans: b
- a) i, i b) $i, -i$ c) $1, -1$ d) $-1, -1$
19. Find c using LMVT for $f(x) = \log x$ in $[1, e]$ Ans: c
- a) $e+1$ b) 0 c) $e-1$ d) 1
20. Find c using CMVT for $f(x) = x^2, g(x) = x^3$ in $[1, 2]$ Ans: d

- a) 14/5 b) 12/7 c) 12/5 d) 14/3
21. The value of c of rolles theorem for $f(x)=\frac{\sin x}{e^x}$ in $(0,\pi)$ is _____. Ans. b
- a) π b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$
22. Find c using CMVT for $f(x)=\sin x$, $g(x)=\cos x$ in $[a, b]$ Ans.d
- a) a-b b) a+b c) (a-b)/2 d) (a+b)/2
23. The value of c of rolles theorem for $f(x)=x+\frac{1}{x}$ in $(\frac{1}{2},2)$ is Ans.b
- a) 0 b) 1 c) 2 d) 3
24. If $u=x^2-2y$, $v = x+y+z$, $w = x-2y+3z$, then $\partial(u,v,w)/\partial(x,y,z) =$ _____. Ans. b
- a) $5x+3$ b) $10x+4$ c) 5 d) 0
25. If $u=x + y + z$, $uv=y + z$, $uvw=z$, then $\partial(x,y,z)/\partial(u,v,w) =$ _____. Ans.c
- a) uv b) u+v c) u^2v d) 1
26. If $u=x+y$, $w=x^3+y^3+z^3$, $v=x^2+y^2+z^2-2xy-2yz-2zx$ are functionally dependent then the relation between them is _____. Ans.a
- a) $u+w=v$ b) 0 c) 1 d) 2
27. If $u=(x-y)/x+y$, $v=xy/(x+y)^2$ are functionally dependent then the relation between them is _____. Ans.a
- a) $u^2+4v=1$ b) $u=v$ c) $u+v=0$ d) none
28. If $u=yz/x$, $v=zx/y$, $w=xy/z$, then $\partial(u,v,w)/\partial(x,y,z) =$ _____. Ans. d
- a) 1 b) 2 c) 3 d) 4

IES

1. The matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is Ans: c
- a) Singular b) Symmetric c) Skew- symmetric d) None
2. The inverse of an orthogonal matrix is Ans: d
- a) Unit matrix b) Hermitian c) Skew Hermitian d) Orthogonal
3. If 2,3,5 are the eigen values of a matrix A and if $B = P^{-1}AP$, then eigen values of B are _
Ans: (a)
- a) 2,3,5 b) -2,-3,-5 c) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ d) $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{5}$
4. The eigen values of $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ are _____. Ans: (b)
- a) 1,3 b) 2,4 c) -3,-4 d) 1,-4
5. The eigen vector of the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$ is _____. Ans: (d)
- a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ d) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

WEBSITES:

1. www.geocities.com/siliconvalley/2151/matrices.html
2. www.mathforum.org/key/nucalc/fourier.html

3. www.mathworld.wolfram.com
4. www.eduinstitutions.com/rec.htm
5. www.isical.ac.in
6. <http://nptel.ac.in/courses/111108066/>
7. <http://nptel.ac.in/courses/111106051/>
8. <http://nptel.ac.in/courses/111102011/>
9. <http://nptel.ac.in/syllabus/syllabus.php?subjectId=111103019>

EXPERT DETAILS:

INTERNATIONAL

Prof. Diliberto, Stephen P.L.

Research Area: Ordinary Differential Equations,

Postal Address: Department of Mathematics

University of California,

Berkeley.

Email ID: diliberto@math.berkeley.edu

NATIONAL

1. Prof. N.S. Gopal Krishna

Research Area: Differential Equations,

Postal Address: Department of Mathematics,

IIT Mumbai.

Email ID : gopal@math.iitb.ac.in

2. Prof. S. Kesavan

Research Area : Analysis and Differential Equations,

Postal Address: Department of Mathematics,

Institute of Mathematical Sciences, Chennai.

Email ID: kesh@imsc.res.in

JOURNALS:

INTERNATIONAL

1. Journal of American Mathematical Society
2. Journal of differential equations - Elsevier
3. Pacific Journal of Mathematics
4. Journal of Australian Society
5. Bulletin of "The American Mathematical Society"
6. Bulletin of "The Australian Mathematical Society"
7. Bulletin of "The London Mathematical Society"

NATIONAL

1. Journal of Interdisciplinary Mathematics
2. Indian Journal of Pure and Applied Mathematics
3. Indian Journal of Mathematics
4. Proceedings of Mathematical Sciences
5. Journal of Mathematical and Physical Sciences.
6. Journal of Indian Academy and Sciences

LIST OF TOPICS FOR STUDENT SEMINARS:

1. Unitary and orthogonal matrices
2. Eigen values and Eigen Vectors
3. Maxima and Minima of functions of two variables
4. Mean value theorems for single variable
5. Concept of sequence, series and alternative series

CASE STUDIES / SMALL PROJECTS:

1. Describe about the Quadratic forms and its nature.

2. Discuss about the Concept of Maxima and Minima of functions of two variables in detail.
3. Describe about the geometrical meaning of mean value theorems.
4. Discuss about the Cayles Hamilton theorem with examples.
5. Describe about Maxima and minima of functions of two variables and three variables using Method of Lagrange.